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A CORRELATION OF CONVECTIVE HEAT TRANSFER AND RECOVERY TEMPERATURE DATA FOR CYLINDERS IN COMPRESSIBLE FLOW†

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Abstract—A correlation is made of heat-transfer data for circular cylinders of high thermal conductivity in the Reynolds number range $2 \times 10^{-2} < Re_o < 10^3$ and the Mach number range $M > 0{\cdot}2$. The Nusselt number is found to vary monotonically from the continuum heat-transfer law $Nu \sim Re^{1/2}$ at high Reynolds numbers to $Nu \sim Re$ in free-molecule flow. The normalized recovery temperature is found to be a function only of the free-stream Knudsen number for Mach numbers greater than 0·6, and a method of estimating the recovery temperature for all Mach numbers is proposed. Empirical formulas are presented for the Nusselt number, recovery factor, and slope of the *Nu–Re* correlation. The results indicate a free-molecule flow energy accommodation coefficient near unity.

NOMENCLATURE

c_p, c_v,	specific heats;
d,	cylinder diameter;
$f(s_1)$, $g(s_1)$,	tabulated functions of s_1;
h,	convective heat-transfer coefficient, $q/(T_w - T_{aw})$;
k,	thermal conductivity;
Kn_∞,	Knudsen number, λ_∞/d;
M,	Mach number, $u_\infty/(\gamma RT_\infty)$;
Nu,	Nusselt number, hd/k;
Pr_o,	Prandtl number, $c_p\mu_o/k_o$;
q,	convective heat-transfer rate;
R,	gas constant;
Re,	Reynolds number, $\rho ud/\mu$;
s_1,	molecular speed ratio, $(\gamma/2)^{1/2}M$;
T_w,	cylinder temperature;
T_{aw},	cylinder temperature for no heat transfer;
u,	velocity;
α,	energy accommodation coefficient;
γ,	specific heat ratio, c_p/c_v;
η,	recovery ratio, T_{aw}/T_o;
$\bar{\eta}_*$,	normalized recovery ratio, $(\eta - \eta_c)/(\eta_f - \eta_c)$;
λ,	molecular mean free path;
μ,	viscosity;
ρ,	density;
τ,	overheat, $(T_w - T_{aw})/T_{aw}$.

Subscripts

$(\ )_o$,	stagnation conditions;
$(\ )_\infty$,	free stream conditions;
$(\ )_c$,	limiting value, continuum flow;
$(\ )_f$,	limiting value, free-molecule flow.

INTRODUCTION

HIGH-ALTITUDE flight and low-pressure laboratory experiments are two increasingly important fields of endeavor where the ambient molecular mean free path λ_∞ may be comparable to a typical body dimension d. Although a number of theoretical analyses are available for the prediction of heat transfer to surfaces in free molecule flow ($\lambda_\infty \gg d$) and in the continuum regime

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($\lambda_\infty \ll d$), the transition between these two limits must be found by experiment at the present time.

In low-speed continuum flow, there have been several correlations of the convective heat-transfer rate between a gas and a circular cylinder as a function of the properties of the flow. McAdams [1] found a successful correlation between the Nusselt number (hd/k) and Reynolds number ($\rho ud/\mu$) by evaluating k and μ at the mean temperature $T_m = 1/2\,(T_w + T_\infty)$, and using the free-stream mass-flow $\rho_\infty u_\infty$ to evaluate the Reynolds number. His correlation was based primarily on the experimental results of Hilpert [2]†, and covered the Reynolds number range $1 < Re < 10^5$. More recently, Douglas and Churchill [3] have offered an improved correlation which accounts for large temperature differences between the cylinder and the stream. At low Reynolds numbers, the experimental results of Collis and Williams [4] agree well with the data of Hilpert for $Re = 1$, and extend the correlation to a value $Re = 0{\cdot}01$. The data of Collis and Williams are in good agreement with the Oseen theory of Cole and Roshko [5] below a Reynolds number of 0·3. For Reynolds numbers greater than 10^4, the work of van Meel [6] may also be consulted.

The results cited above are for continuum flow with a Mach number M close to zero. At higher speeds, the Nusselt number is a function of both Mach number and Reynolds number. Similarly, the recovery temperature T_{aw} is a function of both M and Re. It is the purpose of this paper to review several recent sets of experimental data for the Nusselt number and recovery temperature in compressible flow, and to present empirical formulas which describe these two quantities as a function of M and Re.

An informative correlation of heat-transfer data was compiled by Baldwin, Sandborn, and Laurence [7] in 1960. Their results were based primarily on the transonic experiments of Spangenberg [8] and the subsonic data of Baldwin [9] for $Re > 2$, and on the subsonic experiments of Cybulski and Baldwin [10] in the range $0{\cdot}05 \leq Re \leq 2$. There were several inherent difficulties with this correlation. First, the subsonic data approach the free-molecule theory of Stalder, Goodwin, and Creager [11] only if it is assumed that the energy accommodation coefficient α varies with Mach number, from $\alpha \approx 0{\cdot}5$ at $M \approx 0{\cdot}2$ to $\alpha \approx 0{\cdot}8$ at $M \approx 0{\cdot}9$. In contrast, the more recent investigations in air and argon by Christiansen [12], Dewey [13], and Vrebalovich [14] have indicated an accommodation coefficient near unity. Since no attempt was made in any of these experiments to establish a clean and outgassed surface, an accommodation coefficient near unity would be expected [15, 16]. Observed differences in accommodation coefficient would also suggest differences in the heat-transfer relation in the region between free-molecule and continuum flow.

Second, the normalized recovery temperature variation in the range of Knudsen numbers $Kn_\infty = (\lambda_\infty/d)$ between 0·1 and 10 found by Cybulski and Baldwin [10] in subsonic flow does not agree with the high Mach number correlation given in reference [13]. This was originally ascribed to the difference in Mach number between the two investigations, but the recent experiments of Vrebalovich [14] have shown that the normalized recovery ratio $\bar{\eta}_* = (\eta - \eta_c)/(\eta_f - \eta_c)$ is a unique function of the Knudsen number Kn_∞ for $M \geqslant 0{\cdot}6$. This result suggests a re-examination of the recovery temperature correlation. (It should be emphasized that the recovery temperature in free-molecule flow is independent of the accommodation coefficient.)

Finally, Baldwin [9] and Cybulski and Baldwin [10] report sizeable (up to 30 per cent) non-linear changes in the Nusselt number with overheat τ and with the absolute level of the tunnel stagnation temperature. These changes are in direct contrast to the results of Laufer and McClellan [17], Spangenberg [8], and Dewey [13], where the effects of overheat were found to be small and approximately linear with τ. The experiments of Christiansen [12] were conducted in a shock tube with a cold cylinder and a hot flow; Christiansen's data is in good agreement with the wind-tunnel data of references [13] and [14], again indicating that the effects of overheat are small. The effects of overheat observed by Baldwin [9] and Cybulski and Baldwin [10] may be due to an effect of surface temperature on

† The data of Hilpert [2] are more correctly interpreted in reference [3] than in reference [1].

accommodation coefficient, as Baldwin, Sandborn, and Laurence [7] suggest.

From the above discussion, it is clear that a meaningful correlation cannot be constructed without selecting a portion of the available data. The present correlation is based on the hot-wire heat-transfer data of Christiansen [12], Dewey [13], Laufer and McClellan [17], Spangenberg [8], Vrebalovich [14], Weltman and Kuhns [18], and Wong [19] in compressible flow, and the data of Hilpert [2] and the theory of Cole and Roshko [5] in the incompressible regime. The recovery temperature correlation is based on the data of Dewey [13], Laufer and McClellan [17], Sherman [20], Stalder, Goodwin and Creager [11], and Vrebalovich [14]. Based on an energy accommodation coefficient of unity in free molecule flow, the heat-transfer correlation is believed to be accurate to ± 7 per cent for $M = 0$ and $M > 0{\cdot}2$ over the Reynolds number range $10^{-2} \leqq Re_o \leqq 10^3$. The normalized recovery factor $\bar{\eta}_*$ is believed to be known within $\pm\, 0{\cdot}1$ for $M > 0{\cdot}6$, using the universal relation $\bar{\eta}_* = \bar{\eta}_* \,(Kn_\infty)$. This means an uncertainty in the cylinder recovery temperature of less than ± 2 per cent for *all* free-steam Mach numbers.

HEAT LOSS CORRELATION

The heat-loss data are shown in Fig. 1 in terms of the three parameters

$$Nu_o = (hd/k_o) \equiv \frac{[q/(T_w - T_{aw})]d}{k_o};$$

$$Re_o = \frac{\rho_\infty u_\infty d}{\mu_o}; \quad M_\infty = \frac{u_\infty}{\sqrt{(\gamma R T_\infty)}}, \quad (1)$$

where the subscript $(\ )_o$ refers to evaluation at the free-stream stagnation temperature. These parameters were also used by Baldwin, Sandborn, and Laurence [7] and others [11, 13], and offer distinct advantages over alternative formulations when the cylinders are used as instruments in non-uniform flow fields. The correlation is based on the measured Nusselt number for the cylinder temperature approaching the recovery temperature, although the effect of overheat is small for all data used in this correlation.

The lines for $M > 0$, $Re_o > 40$ are taken from Spangenberg for $M < 2$, and from Laufer and McClellan for $M > 2$. The dashed lines represent the asymptotic slopes $Nu_o \sim (Re_o)^{1/2}$ which are characteristic of continuum high Reynolds

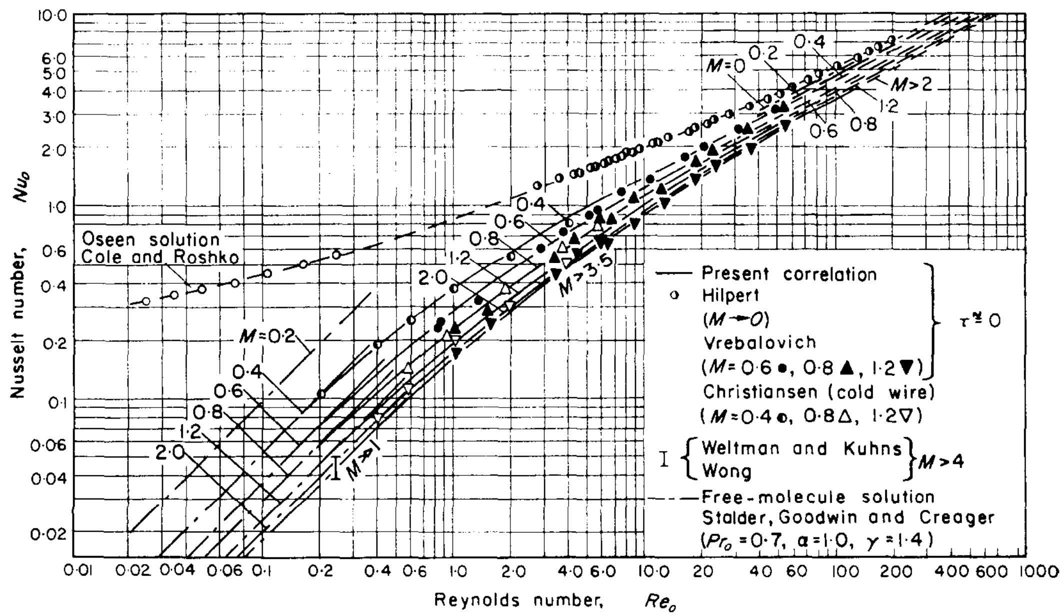

FIG. 1. Empirical correlation of cylinder heat transfer at low Reynolds numbers.

number laminar flow. The line marked $M > 3{\cdot}5$ and $M \gg 1$ in the Reynolds number range $0{\cdot}5 < Re_o < 40$ represents the data of Laufer and McClellan and Dewey, and is believed to be accurate to within ± 3 per cent for the Mach number range indicated. The absence of Mach number dependence in the Nu_o–Re_o relation for $M \gg 1$ is a direct result of the well-known "hypersonic freeze", and is discussed in many recent publications (see e.g. Hayes and Probstein [21]).

The smooth transition between the continuum heat-transfer relation $Nu_o \sim (Re_o)^{1/2}$ and the theoretical free-molecule result is clearly evident. The Knudsen number may be conveniently used to measure the approach to free-molecule flow; from simple kinetic theory.

$$Kn_\infty \equiv (\lambda_\infty/d) = (\pi\gamma/2)^{1/2}\,(M/Re_\infty) \qquad (2)$$

where

$$M = u_\infty/(\gamma R T_\infty)^{1/2} \text{ and } Re_\infty = \rho_\infty u_\infty d/\mu_\infty. \qquad (3)$$

If $Kn_\infty \gg 1$, the free-molecule solution of Stalder, Goodwin, and Creager [11] should apply, so that the Nusselt number is given by

$$Nu_o \equiv (hd/k_o) = \frac{\gamma - 1}{2(\pi)^{3/2}}\,\alpha Re_o Pr_o \left(\frac{g(s_1)}{s_1}\right); \qquad (4)$$

$$Re_o = \rho_\infty u_\infty d/\mu_o;\ Pr_o = c_p \mu_o/k_o; \qquad (5)$$

$$s_1 = (\gamma/2)^{1/2} M, \qquad (6)$$

where $g(s_1)$ is tabulated in reference [11] as a function of the molecular speed ratio s_1 and the number of excited degrees of freedom of the molecules. All of the data shown in Fig. 1 approach the value of Nu_o given by equation (4) with an energy accommodation coefficient α of unity.

The data of Christiansen [12] and Vrebalovich [14] in transonic flow are particularly illuminating. Christiansen performed his experiments in a shock tube, where the cylinder (a fine wire) was maintained at a temperature well below its recovery temperature, while Vrebalovich used a steady-state wind tunnel with cylinder temperatures approaching the recovery temperature. Since the body temperature enters the Nu_o relation in free-molecule flow only through its effect on the accommodation coefficient α, the excellent agreement of these two experiments for $Re_o < 10$ indicates that the correlation given in Fig. 1 should be valid for a wide variety of physical situations. If the measured accommodation coefficient for $Kn_\infty \gg 1$ differs from unity, the heat-transfer coefficients for $Re_o < 10$ will differ from those shown in Fig. 1; the correlation of Baldwin *et al.* [7], on the other hand, was based on a value of α which varied with Mach number as previously discussed.

EMPIRICAL HEAT LOSS FORMULAS

It is useful to express the heat loss correlation of Fig. 1 in analytic form. Because of the wide range of Mach numbers and Reynolds numbers, an accurate representation requires formulas which are, algebraically, somewhat involved. However, the form of the equations is suitable either for hand calculation or high-speed computers, and the accuracy of the expressions is far superior to several of the earlier correlation relations.

The correlation is written in two parts in the following form:

$$Nu_o\,(Re_o; M) = Nu_o\,(Re_o; \infty)\ \Phi\,(Re_o; M); \qquad (7)$$

$$\Phi\,(Re_o; M) = Nu_o\,(Re_o; M)/Nu_o\,(Re_o; \infty). \qquad (8)$$

The first term $Nu_o(Re_o; \infty)$ represents the $M \gg 1$ curve of Fig. 1, and is expressed as

$$\left.\begin{aligned} Nu_o(Re_o; \infty) = Re_o^n \Big[& 0{\cdot}1400 + 0{\cdot}2302 \times \\ & \times \left(\frac{Re_o^{0\cdot7114}}{15{\cdot}44 + Re_o^{0\cdot7114}}\right) \\ + \left(\frac{0{\cdot}01569}{0{\cdot}3077 + Re_o^{0\cdot7378}}\right) & \left(\frac{15}{15 + Re_o^3}\right)\Big], \end{aligned}\right\} \qquad (9)$$

where

$$n = 1 - 1/2\left(\frac{Re_o^{0\cdot6713}}{2{\cdot}571 + Re_o^{0\cdot6713}}\right). \qquad (10)$$

Equations (9) and (10) represent the *mean line* through the data of Dewey [13] and Laufer and McClellan [17] within $\pm 0{\cdot}75$ per cent.† Equation (9) asymptotically approaches the limiting values

† The scatter of these two sets of data is less than ± 5 per cent.

$$\underline{Re_o \to \infty}\; Nu_o\,(Re_o;\, \infty) = 0{\cdot}3702\; Re_o^{1/2} \quad (11)$$

$$\underline{Re_o \to 0}\; Nu_o\,(Re_o;\, \infty) = 0{\cdot}1910\; Re_o \quad (12)$$

$$(Pr_o = 0{\cdot}7,\ a = 1,\ \gamma = 1{\cdot}4).$$

The quantity $n\,(Re_o)$ given by equation (10) is the approximate slope of the Nu_o–Re_o relation for $M \gg 1$, and varies monotonically from 1/2 for $Re_o \gtrsim 200$ to 1 for $Re_o \lesssim 10^{-2}$. For subsonic and transonic flow, the departure of the slope n from the high Reynolds value of 1/2 occurs at a lower value of Re_o.

The function $\Phi\,(Re_o,\, M)$ is a measure of the departure of the Nu_o–Re_o relation from the relation for $M \gg 1$. It is well represented by

$$\Phi\,(Re_o;\, M) = 1 + A(M) \times \left[1{\cdot}834 - 1{\cdot}634\left(\frac{Re_o^{1{\cdot}109}}{2{\cdot}765 + Re_o^{1{\cdot}109}}\right)\right] \times \left[1 + \left(0{\cdot}300 - \frac{0{\cdot}0650}{M^{1{\cdot}670}}\right)\left(\frac{Re_o}{4 + Re_o}\right)\right], \quad (13)$$

where

$$A(M) = \frac{0{\cdot}6039}{M} + 0{\cdot}5701\left[\left(\frac{M^{1{\cdot}222}}{1 + M^{1{\cdot}222}}\right)^{1{\cdot}569} - 1\right]. \quad (14)$$

This expression passes smoothly from the high Reynolds number relation

$$\Phi\,(;\, M0) = 1 + 0{\cdot}200\, A(M)\left[1{\cdot}300 - \frac{0{\cdot}0650}{M^{1{\cdot}670}}\right] \quad (15)$$

to the free-molecule limit

$$\Phi(\infty;\, M) = \frac{0{\cdot}2023}{M} + 0{\cdot}1910\left(\frac{M^{1{\cdot}222}}{1 + M^{1{\cdot}222}}\right)^{1{\cdot}596}. \quad (16)$$

The results of free-molecule theory are tabulated in reference [11]. The fact that the free-molecule heat-transfer law possesses simple closed form solutions as $M \to 0$ and $M \to \infty$ leads to the simple relation given by equation (16). The values $a = 1$, $Pr_o = 0{\cdot}7$, and $\gamma = 1{\cdot}4$ have been used in deriving the latter expression. The differences between the exact solutions of Stalder *et al.* [11] and equation (16) are less than ± 3 per cent for all M, less than $\pm 0{\cdot}5$ per cent for $0{\cdot}8 < M < \infty$, and equation (16) approaches the exact analytic solution as $M \to 0$ and $M \to \infty$.

RECOVERY TEMPERATURE CORRELATION

Two years ago (reference [13]), a correlation was proposed between the normalized recovery temperature $\bar{\eta}_*$ of a conducting cylinder and the free stream Knudsen number Kn_∞. $\bar{\eta}_*$ is defined by

$$\bar{\eta}_* = (\eta - \eta_c)/(\eta_f - \eta_c), \quad (17)$$

where η is the ratio of the cylinder recovery temperature to the free-stream stagnation temperature and the subscripts $(\ )_f$ and $(\ )_c$ denote free-molecular and high Reynolds number continuum limits respectively. The recovery temperature in free-molecule flow is given by the theoretical expression (reference [11])

$$\eta_f = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-1} \frac{f(s_1)}{g(s_1)}. \quad (18)$$

For high Reynolds numbers, η_c has been shown experimentally (e.g. reference [22]) to be a function only of Mach number, decreasing from 1·0 at $M = 0$ to 0·95 for Mach numbers greater than about two.* In free-molecule flow, η_f obeys a similar "hypersonic freeze", and approaches the limiting value $[2\gamma/(\gamma + 1)]$.

The data of Laufer and McClellan [17], Sherman [20], Stalder *et al.* [11], and Dewey [13] have shown that the normalized recovery temperature $\bar{\eta}_*$ is a unique function of the free-stream Knudsen number for $M \gtrsim 2$. This result might be expected from the "hypersonic freeze" evidenced by both the continuum and free-molecule results. On the other hand, the data of Cybulski and Baldwin [10] at subsonic speeds indicated a recovery factor $\bar{\eta}_*$ considerably below the high Mach number results at the same value of Kn_∞.

The recent experiments of Vrebalovich [14] have extended the available recovery temperature data to include the transonic range $0{\cdot}4 < M < 1{\cdot}2$. These data, together with the experimental results for higher Mach numbers, are shown in Fig. 2. It is evident that the variation

* See the Appendix for an experimental verification of the continuum limit $\eta_c = 0{\cdot}95$ for $M \gg 1$.

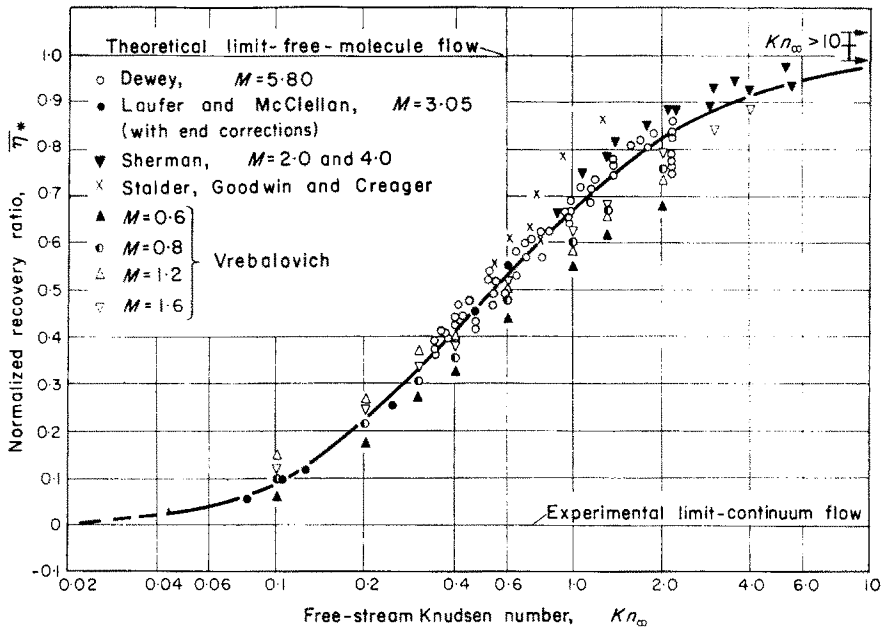

FIG. 2. Normalized variation of recovery temperature with Knudsen number.

of the normalized recovery ratio $\bar{\eta}_*$ is qualitatively and quantitatively similar throughout the complete subsonic–supersonic–hypersonic range. The single curve shown in Fig. 2 agrees with the data for $M \gtrsim 0{\cdot}7$ within $\pm 0{\cdot}1$ in $\bar{\eta}_*$. Since the difference $\eta_f - \eta_c$ rapidly approaches zero as M decreases below one, this curve may be used as an empirical calibration relation for all Mach numbers when computing the recovery temperature. Since the recovery temperature in free-molecule flow is independent of the accommodation coefficient α, it is believed that this correlation is also independent of α.

The recovery temperatures η_f and η_c may be expressed in terms of simple analytic functions. The data correlated by Morkovin [22] may be accurately represented by

$$\eta_c = 1 - 0{\cdot}050 \left(\frac{M^{3\cdot5}}{1{\cdot}175 + M^{3\cdot5}}\right) \qquad (19)$$

and the difference $(\eta_f = \eta_c)$ may be written

$$(\eta_f - \eta_c) = 0{\cdot}2167 \left(\frac{M^{2\cdot80}}{0{\cdot}8521 + M^{2\cdot80}}\right). \qquad (20)$$

Equation (20) is based on the free-molecule calculations of Stalder *et al.* [11] for $\gamma = 1{\cdot}4$. The quantities η_f, η_c, and $(\eta_f - \eta_c)$ given by equations (19) and (20) agree with the exact experimental and theoretical relations to about $\pm 0{\cdot}004$ for all M.

The recovery factor η may be computed using equations (17), (19), and (20) with the values of $\bar{\eta}_*$ shown in Fig. 2. The universal line shown in the figure is represented by the simple expression

$$\bar{\eta}_* = \left(\frac{Kn_\infty^{1\cdot193}}{0{\cdot}4930 + Kn_\infty^{1\cdot193}}\right). \qquad (21)$$

Thus, the recovery ratio $\eta = T_{aw}/T_0$ may be easily computed for all values of Mach number and Reynolds number with a maximum uncertainty of about $\pm 1{\cdot}5$ per cent.

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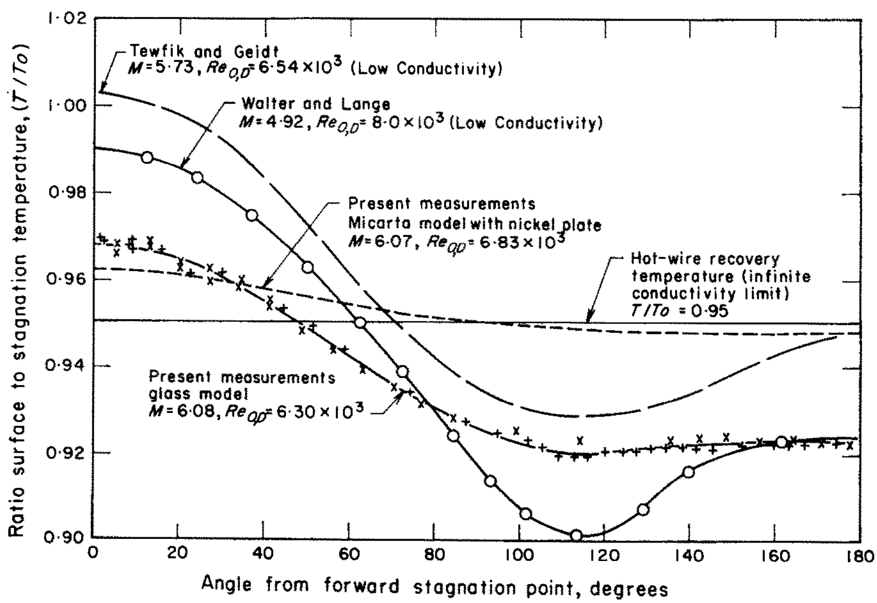

FIG. 3. Surface temperature distribution on an uncooled cylinder.

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APPENDIX

Surface Temperature Distribution on an Insulated Cylinder

Several cylindrical models were built to determine the changes in surface temperature distribution for different model materials. The results of the measurements for two cylinders are compared with the data of Tewfik and Geidt [23] and Walter and Lange [24] in Fig. 3. Surface temperatures were obtained by rotating the cylinder about its axis. The temperature of the nickel-plated micarta model was obtained from an iron-constantan thermocouple imbedded in the nickel plating and polished flush with the surface. For the glass model, a sputtered thin-film platinum resistance gauge was used for the temperature measurement. The resistance–temperature relation of the gauge was determined in a calibration oven.

Figure 3 shows that the surface temperature of a cylinder becomes more uniform as the thermal conductivity of the model increases. The data appear to approach the recovery temperature ratio $\eta_c = (T/T_0) = 0{\cdot}95$ with increasing thermal conductivity of the model. This value is the recovery temperature ratio which has been measured for a fine cylindrical wire in high Reynolds number, high Mach number flows (see Morkovin [20]).

Zusammenfassung—Für die Wärmeübergangswerte von Zylindern mit kreisförmigem Querschnitt und mit hoher Wärmeleitfähigkeit wird eine Beziehung aufgestellt im Bereich der Reynoldszahlen $2 \times 10^{-2} < Re_0 < 10^3$ und der Machzahlen $M > 0{,}2$. Es ergibt sich, dass die Nusseltzahl monoton von dem Wärmeübergangsgesetz im Kontinuum $Nu \sim Re^{1/2}$ bei hohen Reynoldszahlen bis zu $Nu \sim Re$ im Strom freier Moleküle variiert. Die normalisierte Rückgewinn-Temperatur wird als reine Funktion der Knudsenzahl des Freistromes für Machzahlen grösser als 0,6 gefunden. Eine Methode zur Schätzung der Rückgewinntemperatur für alle Machzahlen wird vorgeschlagen. Für die Nusseltzahl, den Rückgewinnfaktor und die Neigung der *Nu–Re*—Beziehung werden empirische Formeln angegeben. Die Ergebnisse zeigen, dass der Energieakkomodationsbeiwert für die freie Molekularströmung nahezu eins ist.

Аннотация—Проводится сопоставление данных по теплообмену для круговых цилиндров с высоким коэффициентом теплопроводности в диапазоне чисел Рейнольдса $2 \times 10^{-2} < Re_0 < 10^3$ и чисел Маха $M > 0{,}2$. Найдено, что число Нуссельта монотонно изменяется от значений, описываемых законом переноса тепла в континууме $Nu \approx Re^{1/2}$ при больших числах Рейнольдса до $Nu \sim Re$ в случае свободно молекулярного течения. Найдено также, что нормализованная температура восстановления является функцией только числа Кнудсена при числах Маха больше 0,6 и предложен метод определения температуры восстановления для всех чисел Маха. Приводятся эмпирические формулы для числа Нуссельта, коэффициента восстановления и наклона корреляционной кривой *Nu–Re*.

Результаты показывают, что коэффициент аккомодации энергии свободно молекулярного потока близок к единице.